Lecture 14

Material and Homework

Read:
Chapter 10 Yariv and Yeh
Chapter 11 Yariv and Yeh

Spontaneous Emission Noise

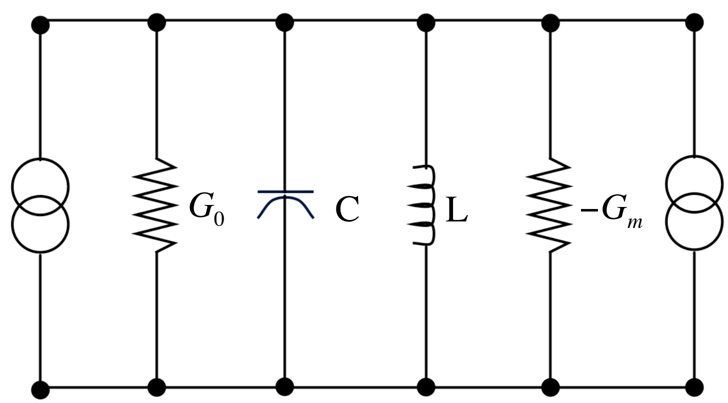
- The spontaneous emission in a laser also contributes to the detected signal to noise ratio (SNR)
  - Photons spontaneously created in the laser cavity will be present at the laser output in addition to stimulated emission.
  - The spontaneous emission and the laser cavity Q will define the laser linewidth

Spontaneous Emission Noise

- We can model the laser cavity as an RLC circuit
  - Conductance $-G_m$ for the gain and G_0 as the loss mechanisms
  - Noise generator $\overline{i_N^2}\Big|_{loss}$ to model Johnson noise due to G_0
  - Noise generator $\overline{i_N^2}\Big|_{SE}$ to model noise due to spontaneous emission for a population inversion represented by temperature T_m is given by $N_2/N_1 = e^{-\hbar\omega/kT_m}$ and $T_m<0$ for $N_2>N_1$
  - Using the Boltzman relation

$$\overline{i_N^2}\Big|_{SE} = \frac{-4\hbar\omega G_m \Delta\omega}{2\pi\left(e^{\hbar\omega/kT_m} - 1\right)} = \frac{4\hbar\omega G_m N_2 \,\Delta\omega/2\pi}{N_2 - N_1}$$

$$\overline{i_N^2}\Big|_{loss} = \frac{4\hbar\omega G_0 \Delta\omega}{2\pi\left(e^{\hbar\omega/kT} - 1\right)}$$



$$\overline{i_N^2}\Big|_{SE} = \frac{-4\hbar\omega G_m \Delta\omega}{2\pi\left(e^{\hbar\omega/kT_m} - 1\right)}$$

Spontaneous Emission Noise

- Defining the quality factor for the equivalent circuit using $\omega_0=(LC)^{-1/2}$

$$\frac{1}{Q}=\frac{1}{Q_0}-\frac{1}{Q_m}=\frac{G_0}{\sqrt{C/L}}-\frac{G_m}{\sqrt{C/L}}=\frac{G_0}{\omega_0^2 C}-\frac{G_m}{\omega_0^2 C}$$

- The voltage across the equivalent impedance due to the current sources is

$$V(\omega)=\frac{i}{C}\frac{I(\omega)}{\dfrac{\omega_0^2-\omega^2}{\omega}+\dfrac{i\omega_0}{Q}}$$

- The total current can be written assuming the sources are uncorrelated and the total is the sum of the mean squared values

$$\overline{|I(\omega)|^2}=4\hbar\omega\left(\frac{G_m N_2}{N_2-N_1}+\frac{G_0}{e^{\hbar\omega/kT}-1}\right)\frac{d\omega}{2\pi}$$

Spontaneous Emission

- The total power output from the oscillator assuming spontaneous emission dominates Johnson noise (Thermal noise)

$$P = G_0 \int_0^\infty \frac{\left|V(\omega)\right|^2}{d\omega} d\omega = \frac{\hbar G_m G_0}{2\pi C^2}\left(\frac{N_2}{N_2 - N_1}\right)\int_0^\infty \frac{\omega d\omega}{(\omega_0 - \omega)^2 + \left(\frac{\omega_0}{2Q}\right)^2} \approx \frac{\hbar G_m G_0 Q}{C^2}\left(\frac{N_2}{N_2 - N_1}\right)\Bigg|_{\omega \approx \omega_0}$$

- In oscillation, the gain equals losses ($G_0 \sim G_m$) and the power can be written as

$$P = \frac{\hbar G_0^2 Q}{C^2}\left(\frac{N_2}{N_2 - N_1}\right)$$

- Observing that the voltage spectral density near resonance has a linewidth $\Delta\omega = \omega_0/Q$, rewriting using the above equation that relates P to Q, we obtain the Schawlow-Townes linewidth that relates the resonant passive cavity linewidth ($\Delta\nu_{1/2}$) to the laser linewidth, noting that $N_2 = F(P)$

$$\Delta\nu = \frac{\Delta\omega}{2\pi} = \frac{\omega_0}{2\pi P} = \frac{\omega_0}{2\pi Q \frac{\hbar G_0^2}{C^2}\left(\frac{N_2}{N_2 - N_1}\right)} = \frac{2\pi h\nu_0 \left(\Delta\nu_{1/2}\right)^2}{P}\left(\frac{N_2}{N_2 - N_1}\right)$$

Laser Phase Noise

- Laser noise comes from
  - Cavity length fluctuations
  - Vibrational and temperature fluctuations
  - Quantum mechanical fluctuations
- The later, quantum mechanical fluctuations, occcurs from the spontaneous emission power being random events that continuously add to the output laser emission field
- Each spontaneous emission event adds a random phase $\Delta\theta$ to the optical (electric) field

$$\Delta\theta = \frac{1}{\sqrt{n}}\cos\phi$$

- After N uncorrelated spontaneous emission events, assuming a uniform distribution of $\Delta\theta$ over $[0,2\pi]$ the total field phase does an angular random walk, averaging over a large number of spontaneous emission events

$$\left\langle\left[\Delta\theta(N)\right]^2\right\rangle = \left\langle\left(\Delta\theta_{\text{one emission}}\right)^2\right\rangle N = \frac{1}{\bar{n}}\left\langle\cos^2\theta\right\rangle N$$